PRELIMINARY VERSION – DO NOT CIRCULATE

A fuzzy logic application to Anti-Money-Laundering supervision[1]

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1. Introduction

According to the Financial Action Task Force (FATF) a risk-based approach (RBA) should be implemented by countries for their anti-money laundering and counter financing terrorism (AML/CFT) regime design. In order to accomplish the RBA principle the Supervising Authorities must adequate their supervision systems by adopting indicators measuring the risk of money laundering (ML). The aim of these indicators is to rank the supervised entities on the basis of the estimated risk exposure in order to deploy the AML action on the financial system in a more targeted way. To this end, since 2012 the Italian financial intelligence unit (UIF) and the Directorate General for Financial Supervision and Regulation of Italy's Central Bank have been cooperating to develop a system of risk-based indicators for financial intermediaries. The present work is a partial result of this fruitful collaboration.

The aim of this document is to show an application of fuzzy logic for the construction of AML indicators for non-banking intermediaries. Its main advantage over traditional methods is the transparency in calculation for the end user (supervisory analyst). To our knowledge, fuzzy logic has been applied to AML supervision for the first time.

2. Literature review

Lofti Zadeh (1965, 1968) describes the fuzzy logic as a new mathematical tool built to overcome the limits of classical logic according to which the truth of a statement has only two values: true or false. The binary representation of the truth of a statement has several limits in practical applications since real world is more complex than that. The fuzzy logic goes beyond the classical one as it does not recognize as valid the Principle of Non-Contradiction (PNC) and the Principle of Excluded Middle (PEM). The goal of fuzzy logic is to create a logical system that resembles and is able to reproduce the way of thinking of a human mind.

The main advantages of fuzzy logic are as follows (Cammarata, 1997):

1. to project, build and up-date a fuzzy application is easier than traditional quantitative methods;
2. end users with little or no quantitative skills can be involved in the application building and can easily interpret the results;
3. the applications are more robust to sudden changes in the context;
4. the fuzzy methodology is considered as a universal approximator (Castro, 1995), which means it can effectively reproduce nonlinear unknown relationships;
5. fuzzy logic is successful in treating complex problems with a more limited computational burden.

Most of the applications of fuzzy logic regard technological fields (Mandel, 1995), but its use has been introduced also in business and finance. Facchinetti *et al.* (2001) show how fuzzy logic achieves a better performance than traditional quantitative methods in assessing customer's credit worthiness, and Shapiro (2005) shows an extended review of fuzzy logic applications in the insurance field. Nevertheless, to our knowledge, there is no relevant applications of fuzzy logic in anti-money laundering field so far.

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3. Methodology

In fuzzy logic a generic statement *A* (e.g., "John is tall") can be assigned any value $\mu(A) \in [0,1]$. This value is called a **degree of truth or membership** and μ is called a **membership function**. In classical logic this value can be only one of the extremes of the range *[0,1]*, i.e., true or false; fuzzy logic can be seen as a generalization of classical logic.

Let us take, for example, the amount of money in a pocket. Suppose the range of this variable is between 0 and 100 euros; in fuzzy jargon, this range is called **Universe of Discourse** and each single value is called a **crisp value** (Mandel, 1995; Cammarata, 1997). We want to define three different sets of amounts in a fuzzy way: e.g., small, medium and high. An example graphical representation of these sets is shown in figure 1. In fuzzy approach, it is possible that a crisp value can belong to two different sets with different degrees of truth as shown in figure 1. Even if it is possible to have complex membership functions, simple polygons, such as triangles and trapeziums, are more commonly used as they are computationally easier to handle with satisfactory results (Cammarata, 1997).

Let x be a crisp value of a generic variable X, $S=\{A_1, A_2, \ldots A_n\}$ a collection of fuzzy sets[3] on the **Universe of Discourse** of X, and $\mu(x \in A_i) \in [0,1]$ the degree of membership of x to the generic set A_i. A fuzzy system is defined as a nonlinear mapping of an input data vector to a scalar output (Mandel, 1995). It is made by three elements (figure 2):

1. Membership function of the input variable (to fuzzify the input);[4]
2. Fuzzy rules;
3. Membership function of the output (to defuzzify the output).[5]

Figure 1. Fuzzy sets for an amount of money.

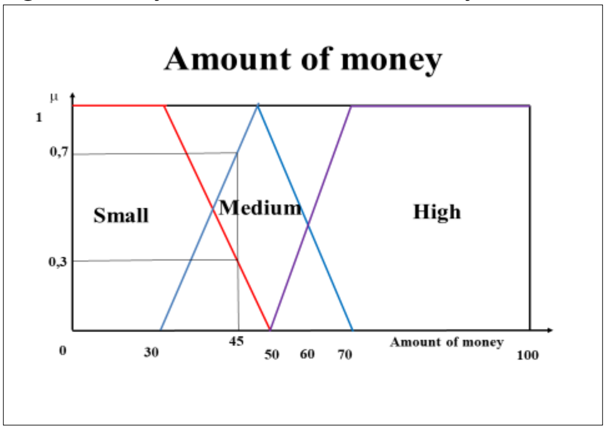
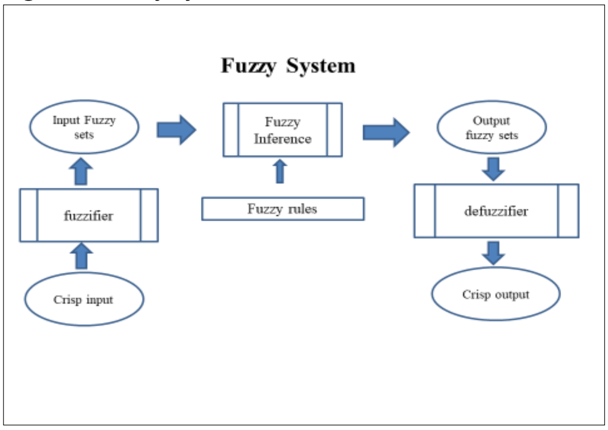


Figure 2. Fuzzy system scheme.



Fuzzy sets can be considered like **Linguistic Variables** (Zadeh, 1975) describing a range of values belonging to a set (with different membership values)[6] in natural language (e.g., *x* is High, *y* is Slow). More linguistic variables together with a logic operator (OR, AND, NOT) form linguistic relations[7] (Cammarata, 1997). In formal way, let *A*, *B* two fuzzy sets of two variables *X* and *Y*, the evaluation of the degree of truth of linguistic relations are made by:

[1] $\mu(x \in A \text{ AND } y \in B) = Min[\mu(x \in A), \mu(y \in B)]$;[8]

[2] $\mu(x \in A \text{ OR } y \in B) = Max[\mu(x \in A), \mu(y \in B)]$;

[3] $\mu(x \text{ NOT} \in A) = 1 - \mu(x \in A)$.[9]

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[3] In our example it would be {Small, Medium, High}.
[4] The number of input variables can be greater or equal to 1.
[5] Without loss of generality, we can set the number of outputs equal to 1.
[6] Considering our example "45 is Small", whose degree of truth is the value of the membership function of 45 to Small.
[7] E.g., *x* is High AND *y* is Slow.
[8] In natural language "*x* is *A* and *y* is *B*".
[9] Other types of fuzzy compositions of fuzzy relations are available. We show the three basic ones that we use in our application.

Fuzzy rules are simple IF-THEN rules with a condition or premise - after IF and before THEN - and a conclusion - after THEN (Mendel, 1995).[10] The condition can have a combination of linguistic variables. In classical logic generic rule "IF *P* THEN *Q"* means "*Q* is true if *P* is true". Therefore, in fuzzy way the degree of truth of *P* is the degree of truth of *Q*. If the condition is composed by more than one linguistic variables (e.g., "IF *P1* AND *P2* THEN *Q"*) the degree of truth of the premise is evaluated according to [1], [2] and [3]. The degree of truth of the premise is denoted with α which indicates the "*strength*" of the rule activated.

The output fuzzy set inferred by a fuzzy rule is defined as follows. Let $\mu(z \in O_i)$ the membership function of a generic fuzzy set O_i of the output variable *Z*, then the membership function defining the inferred fuzzy set of the output is $\mu'(z \in O_i)=min[\mu(z \in O_i), \alpha]$ where *z* belongs to the **Universe of Discourse** of *Z*.

In a fuzzy system with more than one rule, we can have two cases:

1. more than one rule infers the same fuzzy set of output;
2. more rules infer different fuzzy sets of output.

In both cases, we need to compose the inference results. In the first case, the composition of inference of the fuzzy set is quite straightforward: if we have two rules inferring the same fuzzy set with $\mu_{r1}'(z \in O_i)=min[\mu (z \in O_i), \alpha_{r1}]$ and $\mu_{r2}'(z \in O_i)=min[\mu (z \in O_i), \alpha_{r2}]$, the membership function of the inferred output is $\mu'(z \in O_i)=min[\mu (z \in O_i), max(\alpha_{r1}, \alpha_{r2})]$. In the second case the several outputs are composed with logic operation OR (max in fuzzy way): the resulting inferred fuzzy membership function of output is $\mu'(z \in O)=max[\mu(x \in O_1), \mu(x \in O_2), \ldots, \mu(x \in O_n)]$ $\forall z \in O$ supposing that the universe of discourse of *Z* is partitioned in *n* fuzzy sets O_i.[11]

Finally, in order to convert the inferred output into a single point value (crisp value) we need to apply a defuzzification. Several defuzzification techniques can be found in the literature (Mendel, 1995) and we chose to apply the method of Center of Gravity (CoG)[12]. Let $\mu'(z)$ the membership function of the inferred output, the crisp value of the output will be $U = \frac{\int_{min}^{max} z\mu'(z)dz}{\int_{min}^{max} \mu'(z)dz}$.

4. Data description and building of the fuzzy system

(a) Data

The source of data used in our analysis is the aggregate anti-money-laundering reports (S.AR.A. from the Italian acronym) database. Under the Italian anti-money laundering law (Legislative Decree no. 231/2007 and subsequent amendments), banks and other financial intermediaries mandatorily report on a monthly basis to Italy's Financial Intelligence Unit (UIF) all transactions equal or over 15.000 euros, after aggregating them by branch, customer sector and type of transaction.

Our systems of fuzzy indicators are implemented on 2019 S.AR.A. data for non-banking intermediaries , which are split in different classes according to their main activity (eg, stock brokers, payment institutes, etc.). Each class is further divided into sub-groups depending on additional features, such as specialization and scale of activity. In what follows we will refer to a class of 192 intermediaries, which is split into eight homogeneous groups, each one having its own fuzzy system.

A set of 11 indicators, grouped into three categories (Table 1), is employed on a differentiated basis according to the group of intermediaries.[13]

[10] For example: IF speed is High THEN braking_distance is Long.

[11] Other compositions of output are possible; see Cammarata (1997) for further details.

[12] We used the software module (Jfuzzylogic) which approximates the integral on 1.000 equispaced sample points: $U = \frac{\sum_1^{1000} \mu(u_i)u_i}{\sum_1^{1000} \mu(u_i)}$.

[13] The selection of the indicators was made involving AML supervisor experts and taking into account the characteristics of the available data.

Table 1. Indicators

Category	Indicator
Wire transfers	International wire transfers (euros) **(a)** Wire transfers from and to risky countries (euros) **(b)** Domestic wire transfers (euros) **(c)** Total wire transfers (euros) **(d=a+c)** Share of domestic wire transfers on total wire transfers for every 1,000 euros **e=c/(a+c)** Share of wire transfers from and to risky countries on total wire transfers for every 1,000 euros **f=(b/a)**
Dimension	Total amount of reported transactions (euros) **(t)** Number of transactions **(n)** Average amount per transaction (euros) **(m=t/n)**
Type of customers	Total amount of operations ordered by households (euros) **(o)** Share of households' operations on total amount of transactions for every 1,000 euros **v=(o/t)**

(b) Building of the fuzzy systems

In order to build a fuzzy system it is necessary to decide the fuzzification of the input variable (the indicators), the rules, an output variable and its defuzzification. For level indicators (a), (b), (c), (d), (t), (n), (m) and (o) non-zero[14] values in 2017-2019 time period are employed to compute the percentiles, 10°, 30°, 50°, 70°, 90°, which are used for fuzzification (figure 3). For share indicators (e), (f) and (v) we adopt the fuzzification showed in figure 4. The output variable is a risk exposure score ranging from 0 to 1000 (see figure 5).

Figure 3. Fuzzification of a non-negative continuous input variable[15]

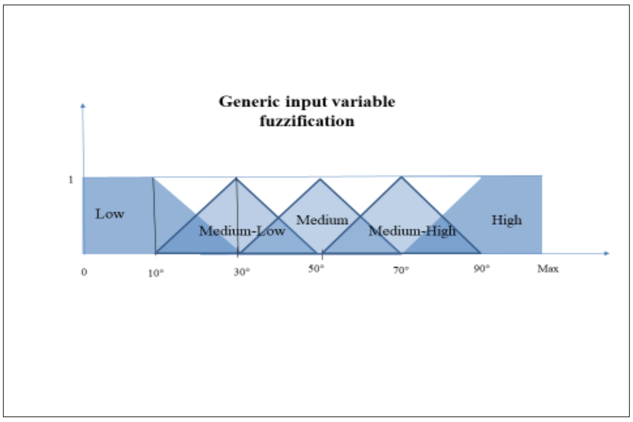
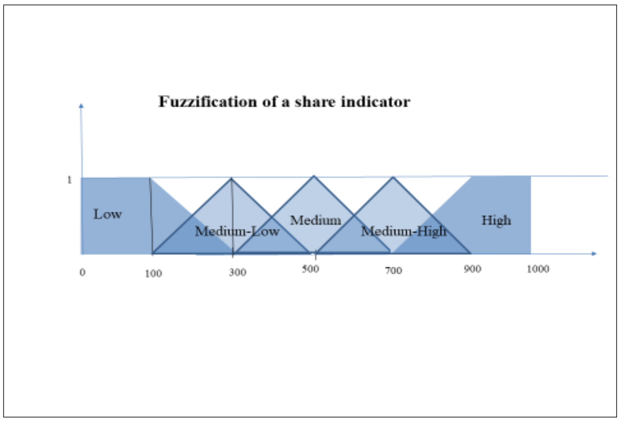


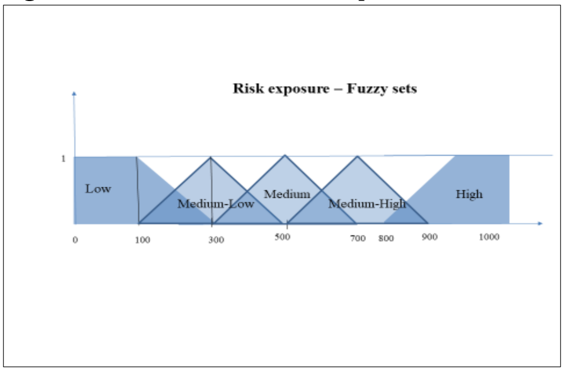
Figure 4. Fuzzification of share indicators



Finally, the CoG method is employed to defuzzify the output variable, delivering defuzzified output values ranging from 0 to 1,000. A synthetic integer rating score (from 1 to 4) is assigned as a function of the final risk exposure score.

[14] Because of zero proliferation in our data-set we are obliged to skip zeroes to have non-zero percentiles. To deal with a small number of observations or particular cases ad hoc solutions are used.
[15] The maximum value is an arbitrary value to approximate infinite.

Figure 5. Fuzzification of the output variable



5. Results

The distribution of the assigned rating coming from the fuzzy systems is unbalanced across the intermediaries (Table 2): only 20% intermediaries have a top rating (3 or 4), and modal rating is 1 with 61% of intermediaries. Rating delivered by the fuzzy systems takes into account only quantitative aspects of the matter. A further step into a qualitative analysis will be required in order to assign an ultimate rating to the intermediaries.

Table 2. Distribution of classified intermediaries by rating - 2019

Rating	**N.**	**%**
1	117	61
2	36	19
3	27	14
4	12	6
Total	192	100

6. Conclusions

The implementation of our fuzzy systems has shown the possibility to apply soft computing techniques in AML supervision. The AML analysts involved in the building of the fuzzy systems have expressed a general positive feeling about the new method. In particular, the transparency in calculation is very appreciated. Results also confirm the importance to involve experts in designing the systems, even if they have no or limited quantitative skills.

In the next few months, the fuzzy systems will be released for the other groups of non-banking intermediaries, thus completing the fuzzy indicator systems for all non-banking intermediaries. An extension of the methodology to banking intermediaries will be eventually considered.

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